# 2.5 MOST EFFICIENT CROSS SECTION

- For steady uniform flow down a channel we developed the following relationship:

$$\tilde{u} = \frac{\kappa}{n} R_H^{2/3} S_0^{1/2} \tag{2.5.1}$$

$$\text{where } \kappa = \text{unit coefficient} = 1.486 \text{ for USCS units}/1.000 \text{ for SI units} \tag{2.5.2}$$

$$n = \text{Manning coefficient} \tag{2.5.3}$$

$$R_H = \text{Hydraulic radius} \tag{2.5.4}$$

$$S_0 = \text{bottom slope of the channel} \tag{2.5.5}$$

- Total flow down the channel is

$$Q = \frac{\kappa}{n} A R_H^{2/3} S_0^{1/2} \tag{2.5.6}$$

$$\text{where A} = \text{cross-sectional area of water} \tag{2.5.7}$$

- For a given area of water cross section, $Q$ will be maximum when $R_H$ is maximum.

- Since $R_H = \dfrac{A}{P_w}$

$$Q = \frac{\kappa}{n}\frac{A^{5/3}}{P_w^{2/3}}S_0^{1/2} \tag{2.5.8}$$

- Thus for a given $n$, $A$ and $S_0$, $Q$ will be a maximum when $P_w$ is a minimum. The corresponding cross section will be the most efficient cross section.
- A circle has the least perimeter for a given area of any geometric shape.

$$R_H = \frac{\pi r^2}{2\pi r} = \frac{r}{2} \tag{2.5.9}$$

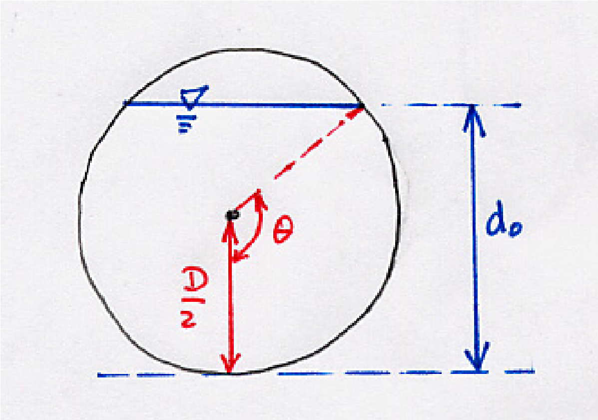
- A semi-circle has

$$R_H = \frac{\left(\dfrac{\pi r^2}{2}\right)}{\pi r} = \frac{r}{2} \tag{2.5.10}$$

- Semi-circular open channel will discharge more water than any other shape (assuming that the area, slope and surface roughness are the same).
- Semi-circular shape/circular shape are practical for concrete and steel pipes.
- Canals excavated in earth must have a trapezoidal shape with slopes less than the ***angle of repose*** of the saturated bank material.
    - Thus canal design is based partly on bank stability considerations.

## **Circular Section Not Flowing Full**

- Consider the following circular cross section under steady uniform flow conditions:




- Area of water can be computed as:

$$A = \frac{D^2}{4}(\theta - \sin\theta\cos\theta) \tag{2.5.11}$$

$$A = \frac{D^2}{4}\left(\theta - \frac{1}{2}\sin 2\theta\right) \tag{2.5.12}$$

where $D$ = pipe diameter (2.5.13)

$\theta$ = angle in radians (2.5.14)

- Wetted perimeter is

$$P_w = D\theta \tag{2.5.15}$$

- The hydraulic radius is computed as

$$R_H = \frac{A}{P_w} \tag{2.5.16}$$

$$R_H = \frac{D}{4}\left(1 - \frac{\sin 2\theta}{2\theta}\right) \tag{2.5.17}$$

- Recall that for steady uniform flow

$$Q = \frac{\kappa}{n} A R_H^{2/3} S_0^{1/2} \tag{2.5.18}$$

- In order for Q to be maximum, $AR_H^{2/3}$ must be a maximum.

- Define $F$ as

$$F = AR_H^{2/3} \tag{2.5.19}$$

$$F = \frac{D^2}{4}\left(\theta - \frac{1}{2}\sin 2\theta\right)\left[\frac{D}{4}\left(1 - \frac{\sin 2\theta}{2\theta}\right)\right]^{2/3} \tag{2.5.20}$$

- Maximum value of $F$ is found as follows:

$$\frac{dF}{d\theta} = 0 \tag{2.5.21}$$

- Solving for $\theta$ we find:

$$\theta = 151.2^0 \tag{2.5.22}$$

- This corresponds to

$$d_0 = 0.938D \tag{2.5.23}$$

- Thus the maximum discharge $Q$ occurs at partial depth.
- Note that for a given area $A$, the semi-circle would still allow for more flow.