2.5 MOST EFFICIENT CROSS SECTION

• For steady uniform flow down a channel we developed the following relationship:

\[ u = \frac{K}{n} R_{H}^{2/3} S_{0}^{1/2} \]  
(2.5.1)

where \( K = \) unit coefficient = 1.486 for USCS units/1.000 for SI units  
(2.5.2)

\[ n = \text{Manning coefficient} \]  
(2.5.3)

\[ R_{H} = \text{Hydraulic radius} \]  
(2.5.4)

\[ S_{0} = \text{bottom slope of the channel} \]  
(2.5.5)

• Total flow down the channel is

\[ Q = \frac{K}{n} A R_{H}^{2/3} S_{0}^{1/2} \]  
(2.5.6)

where \( A = \) cross-sectional area of water  
(2.5.7)
• For a given area of water cross section, \( Q \) will be maximum when \( R_H \) is maximum.

• Since \( R_H = \frac{A}{P_w} \)

\[
Q = \frac{kA^{5/3}}{n^{2/3}P_w^{2/3}}S_0^{1/2}
\]  

(2.5.8)

• Thus for a given \( n, A \) and \( S_0 \), \( Q \) will be a maximum when \( P_w \) is a minimum. The corresponding cross section will be the most efficient cross section.

• A circle has the least perimeter for a given area of any geometric shape.

\[
R_H = \frac{\pi r^2}{2\pi r} = \frac{r}{2}
\]  

(2.5.9)

• A semi-circle has

\[
R_H = \frac{\left(\frac{\pi r^2}{2}\right)}{\pi r} = \frac{r}{2}
\]  

(2.5.10)
• Semi-circular open channel will discharge more water than any other shape (assuming that the area, slope and surface roughness are the same).

• Semi-circular shape/circular shape are practical for concrete and steel pipes.

• Canals excavated in earth must have a trapezoidal shape with slopes less than the *angle of repose* of the saturated bank material.

  • Thus canal design is based partly on bank stability considerations.
Circular Section Not Flowing Full

• Consider the following circular cross section under steady uniform flow conditions:

• Area of water can be computed as:

\[ A = \frac{D^2}{4} (\theta - \sin \theta \cos \theta) \]  
\[ (2.5.11) \]

\[ A = \frac{D^2}{4} \left( \theta - \frac{1}{2} \sin 2\theta \right) \]  
\[ (2.5.12) \]

where \( D \) = pipe diameter  
\[ \theta = \text{angle in radians} \]  
\[ (2.5.13) \]  
\[ (2.5.14) \]
• Wetted perimeter is

\[ P_w = D\theta \quad (2.5.15) \]

• The hydraulic radius is computed as

\[ R_H = \frac{A}{P_w} \quad (2.5.16) \]

\[ R_H = \frac{D}{4} \left(1 - \frac{\sin 2\theta}{2\theta}\right) \quad (2.5.17) \]

• Recall that for steady uniform flow

\[ Q = \frac{k}{n} A R_H^{2/3} S_0^{1/2} \quad (2.5.18) \]

• In order for \( Q \) to be maximum, \( AR_H^{2/3} \) must be a maximum.
• Define $F$ as

$$F = AR_H^{2/3}$$

(2.5.19)

$$F = \frac{D^2}{4}\left(\theta - \frac{1}{2}\sin 2\theta\right)\left[\frac{D}{4}\left(1 - \frac{\sin 2\theta}{2\theta}\right)\right]^{2/3}$$

(2.5.20)

• Maximum value of $F$ is found as follows:

$$\frac{dF}{d\theta} = 0$$

(2.5.21)

• Solving for $\theta$ we find:

$$\theta = 151.2^0$$

(2.5.22)

• This corresponds to

$$d_0 = 0.938D$$

(2.5.23)

• Thus the maximum discharge $Q$ occurs at partial depth.

• Note that for a given area $A$, the semi-circle would still allow for more flow.