Consider the following ordinary differential equation (ODE) of $u(x)$:

$$
\frac{d^2u}{dx^2} + x \frac{du}{dx} + \frac{u}{6} = -3,
$$

for the domain $0 \leq x \leq 10$

Develop a code using the symmetrical weak weighted residual form of the Galerkin method to solve this ODE on the domain $0 \leq x \leq 10$ using polynomial basis functions. Follow these steps:

a) Determine the essential and natural boundary conditions.

b) Establish the fundamental weak weighted residual form.

c) Using integration by parts, determine the symmetrical weak weighted residual form.

d) Set up the boundary function, $u_B$, and the approximating functions, $\phi_i$, to satisfy the weak admissibility and completeness criteria for the weak weighted residual forms of the Galerkin method.

e) Develop formulae for the entries in the system of simultaneous equations and the right hand side vector to solve using the symmetrical weak weighted residual form.

f) Using Matlab, or other suitable language, develop a code to solve the system for 2, 10, 20 and 100 term expansions. Determine the error for each solution by comparing with the point based analytical solution provided on the course website (Prob_1.soln.txt). Plot your solutions and your errors. Create another figure comparing the $L_2$ error for each of the expansion levels, be sure to use a logarithmic scale along the vertical ($y$) axis.

g) Comment on how the weak Galerkin method compares to the full (strong) form solutions from HW#1. Discuss properties such as convergence, accuracy, computational cost, constraint on basis functions.